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Eccentricity and the Ferry-Porter law

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Is the flicker limit governed by the Ferry-Porter law or by some other law? We previously obtained strong evidence for the wide applicability of the Ferry-Porter law and for its variation with eccentricity [J. Opt. Soc. Am. A 7, 743 (1990)]. Raninen and others [Vision Res. 28, 785 (1988); 31, 1875 (1991)] have questioned our analysis of the data and our conclusions. We show that their criticism of the Ferry-Porter law is based on spurious analyses and that their own data support the Ferry-Porter formulation. We also provide further evidence that the slope of the Ferry-Porter function increases markedly with eccentricity, implying that the inherent temporal properties of the retina grow more rapid with distance from the fovea.

In a psychophysical study of the temporal properties of local regions of the human retina, Tyler and Hamer¹ found strict adherence to the Ferry-Porter law both in the fovea and in the periphery. The Ferry-Porter law² states that critical flicker fusion (CFF) increases directly with the logarithm of stimulus illuminance. Raninen *et al.*³ propose a new principle governing the effects of field size and illuminance on CFF as a function of eccentricity for red light of stable spectral composition. This experiment controls for the variation in spectral composition with illuminance in their earlier study,⁴ in which illuminance was varied by attenuation of a green phosphor with Wratten filters of various colors.

The nature of the temporal limits to visual function is important in many arenas, from temporal resolution to motion perception, the temporal interactions in acuity and spatial vision in general. It is therefore relevant to establish the principles underlying those limits as clearly as possible. The paper by Raninen *et al.*³ contains a number of inaccuracies and misrepresentations of our work, which are used to support the conclusions that they have drawn. In addition to clarifying these inaccuracies, the present Communication comments on how their methodology and data interpretation further affect the interpretation of their study. We also present new CFF data that are relevant to the issues at hand.

In summary, our view is that their analyses do not test whether CFF is governed by the Ferry-Porter law in local retinal regions. Instead, their data are tested against a different law that is incommensurate with the original formulation by Porter.² A proper analysis of their data, on the other hand, reconfirms the Ferry-Porter law and the fact that its slope varies with eccentricity.

THE EFFECT OF FLUX ON CRITICAL FLICKER FUSION

The main solecism of the Raninen *et al.*³ paper is that they perform their analysis on a deviant definition of the Ferry-Porter law (and other laws). The function they de-

rive as best fitting their data therefore cannot be said to depart from the Ferry-Porter law in its original definition. The Ferry-Porter law holds that, above the threshold illuminance I_0 , CFF increases as the logarithm of the illuminance I :

$$\text{CFF} = k(\log I - \log I_0), \quad (1)$$

where $I > I_0$ and with k having a typical value of between approximately 10 and 30 Hz/decade.

Raninen *et al.*³ define their own law in which CFF is raised to a power λ and I is replaced by F , their version of the luminous flux of the stimulus:

$$\text{CFF}^\lambda = k \log F + c, \quad (2)$$

where

$$F = IA_r. \quad (3)$$

Here c is a constant and I , the retinal illuminance, is scaled by A_r , Ricco's area, or the area of linear energy integration, in degrees squared. Raninen *et al.* calculate the value of this F -scaling constant by an equation relating the radius R of Ricco's area to retinal eccentricity E , having the form

$$R = aE^3 + bE + d, \quad (4)$$

where a , b , and d are constants. The result is a complicated expression relating CFF to F -scaled retinal illuminance that is based on a sixth-order function of retinal eccentricity:

$$\text{CFF} = \{k \log[\pi I(aE^3 + bE + d)^2] + c\}^{1/\lambda}. \quad (5)$$

Raninen *et al.* then combine the effects of variation in illuminance, eccentricity, and area on a single graph (Fig. 3 of Ref. 3), in which stimulus area also is scaled with eccentricity according to their estimate of the appropriate magnification factor (M scaling). They attempt to fit these combined data with a single function, which they represent as testing the Ferry-Porter law (when the parameter $\lambda = 1$; for $\lambda = 0.5$, they purport to be testing the Kelly square-root law).

Now, luminous flux obviously is not the same as illuminance, in that flux contains the additional factor of stimu-

lus area. For CFF to show the same relation to flux as to illuminance under variations in area, the sensitivity variation as a function of area would have to be linear. In fact, it is well known that CFF varies more closely with the square root of area, except for very small areas approaching the optical limits of the eye.⁵⁻⁷ We have analyzed this relation and verified that the square-root relation holds for CFF over a wide range of illuminances and peripheral test-field sizes (Ref. 1, Fig. 5). It follows that CFF cannot simultaneously be proportional to $I^{0.5}$ and obey the relation proposed by Raninen *et al.*³ [Eq. (5) above]. In fact, the cited data¹ adhered to the Ferry-Porter law (linear with $\log I$) with an invariant slope of 19 Hz/decade over a thousandfold range of field sizes.

The F scaling of the retinal illuminance by Raninen *et al.* imposes a steep positive curvature on the Ferry-Porter equation since it increases as a sixth-order function of retinal eccentricity. This may be viewed as an attempt to model the change in Ferry-Porter slope with eccentricity. But it is certainly not a test of the local validity of the Ferry-Porter, the Kelly, or the Collins log-log laws.

The data that Raninen *et al.*³ present in support of their hypotheses are weak in three respects. First, the average variance remaining unexplained by their hypothesis is approximately 10%. This may seem like a small amount for behavioral data, but it is large compared with the difference in the predictions between the two hypotheses. It is also a much larger error variance than the fits to the Ferry-Porter law obtained by Tyler and Hamer,¹ in which the residual variance had a mean of 0.2% of the total variance. (The latter was calculated by computing the correlation r of CFF against log retinal illuminance for each observer. The square root of $1 - r^2$ for each observer was calculated. The square of the average of these residual standard deviations constituted an estimate of the average residual variance for the three observers.)

Second, the fits obtained by Rainen *et al.* implicated two different laws for the two observers whom they tested: a Kelly-type 0.3 power law for one and a Collins-type log-log law for the other. This ambiguous result leaves the fundamental basis for the flicker limits unclear in their formulation.

Third, the deviations from the flux hypothesis in Fig. 2 of Ref. 3 were not random but showed a displaced series of segments corresponding approximately to the square root of CFF/area behavior for each eccentricity. What is not obvious from these plots (or those of Fig. 1 of Ref. 3) is the degree to which these data support the Ferry-Porter law. This correspondence is now shown in Fig. 1, where we replot their data for the narrow-band red light (subject RF) at five representative eccentricities (0°, 10°, 22°, 40°, and 70°). The data for these (and the remaining) eccentricities show good conformity to the fitted straight lines, which correspond to the Ferry-Porter function, with error variances averaging 1.2% of the total variance in the data across the nine eccentricities. As recognized by the present authors, the slope of the fitted function increases from 9.2 Hz/decade at 0° to 23 Hz/decade at 40°, confirming that the slope of the CFF-intensity function approximately doubles with this change in eccentricity.¹

The data for the other subject, which were collected by viewing through a Wratten 92 filter, do not appear to conform to a smooth function at any eccentricity. Although

the use of the deep-red filter may make it seem unlikely, the form of the data was consistent with the presence of a scotopic component dominating the CFF response.³ However, in their experiment there was no attempt to suppress the rod contribution to the responses by a masking background or by a suppressive surround. The implication is that the CFF values for subject AR are mediated by two separate receptor systems across the luminance range tested, which would invalidate any attempt to fit the full set of data with a uniform function. Moreover, the upper part of the data was usually represented by a single point, obviating the possibility of fitting the upper range separately.

MISINTERPRETATION OF STATISTICAL ANALYSES

In addition to the contradictions present in the analysis of their own data, Raninen *et al.*³ attempt a direct refutation of the analyses presented by Tyler and Hamer.¹ Although we reported support for the Ferry-Porter law at significance levels that far exceeded the levels provided in statistical tables, they present a reanalysis with no attempt to determine whether there might have been an error in their reading of the data. In fact, there are two such errors.

The first error is that they analyze only the data of our Fig. 4 (Ref. 1), in which we presented supplementary support for the hypothesis with two additional observers; but they ignore the data for the main observer in our Fig. 3 (Ref. 1), which had 22 observations. We found a better fit for the Ferry-Porter law than for the Kelly square-root law by three different methods, which showed F ratios of 182, 180, and 254 (where a ratio of 10.9 would indicate significance at $p < 0.001$). These F ratios implied extreme significance levels far beyond the tabulated levels, although we stated only that the significance exceeded 0.01, because this is held to be more rigorous statistical practice.

The second error is that Raninen *et al.* misread the number of data points in our Fig. 4 (Ref. 1). There is a

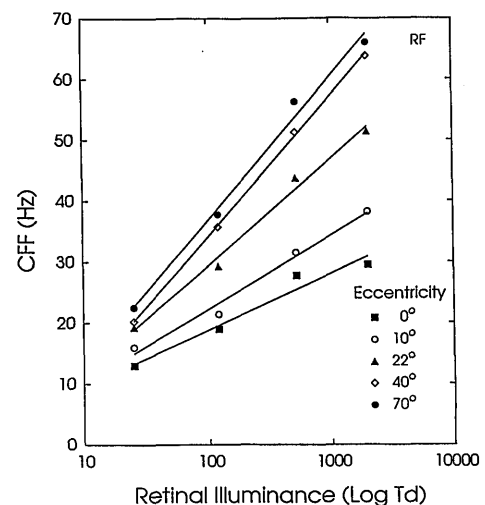


Fig. 1. Plots of CFF versus log retinal illuminance derived from Fig. 1 of Raninen *et al.* (Ref. 3) at five illustrative eccentricities (0°, 10°, 22°, 40°, and 70°). Note the excellent fit of the Ferry-Porter functions (straight lines), with a pronounced increase in slope with eccentricity.

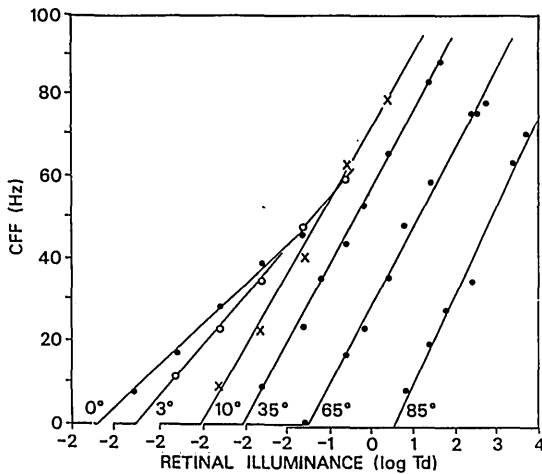


Fig. 2. Data of CFF plotted against log retinal illuminance for 642-nm light for observer RDH at six eccentricities. Abscissa is a multiple axis with a leftward shift of one log unit for each successive eccentricity for clarity. Note the unbiased fit of the Ferry-Porter functions (straight lines), with a pronounced increase in slope with eccentricity.

measured datum point at 0 CFF for each observer, which we obtained by measuring absolute threshold for a static target. It is evident that Raninen *et al.* do not include this in their analysis, since they give the number of data points measured for each observer as seven, when it was in fact eight. Had they included the 0 CFF point, they would have found that the residual errors of the Ferry-Porter fit were both random and normally distributed (or at least did not significantly depart from these properties). They also would have found that either of the statistics that they applied would have shown a highly significant support for the Ferry-Porter law over the alternatives suggested, as did the statistics that we provided.

SUPPORTING DATA

To verify the change in slope of the Ferry-Porter law with eccentricity, we include some data collected in our own laboratory that highlight the effect. The testing conditions were similar to those in a previous study¹ and so are described only briefly here. The stimulus of 25 LED's with a dominant wavelength of 642 nm,⁸ projected through a diffuser so as to give a uniform red stimulus, was set in a hemispheric white surround of equal photopic illuminance. These stimulus conditions maintained light adaptation at a constant level, eliminated any rod contribution to the flicker sensitivity, and ensured that the response was dominated by the R-cone pathway.^{1,9}

The test stimuli were presented so as to stimulate retinal regions that were uniform in structure. The stimulus for central viewing was therefore chosen to be 0.5° in diameter to stimulate the homogeneous central foveola. Peripheral stimuli were placed on the horizontal temporal-field meridian at eccentricities of 3°, 10°, 35°, and 85°. Peripheral stimulus size S was set so as to equate the number of cones stimulated in each test field on the basis of the power function of eccentricity E . The actual equation^{10,11} was

$$S = A\{[1 - w[1 - \cos(0.5\pi E)]]\}/m_c(B + E)^{-0.33}, \quad (6)$$

where $A = 30,000$ cones/mm², $B = 0.2^\circ$, $w = 0.19$, and $m_c = 1.25$.

The sinusoidal flicker modulation was presented in a raised cosine temporal envelope of 1-s duration, so that flicker could not be detected by transients at the beginning or the end of the presentation. CFF values were measured by a yes/no forced-choice staircase method, with equal probability of stimulus or no stimulus on each trial. Three CFF estimates were obtained under each condition tested. The observers' pupils were dilated with 1% Mydracil to hold them constant at all illuminance levels, which were controlled by means of Wratten 96 neutral-density filters mounted in a light-tight mask worn over the eyes. The right eye viewed the stimulus through the appropriate filter density, while the other eye was occluded and in darkness.

For peripheral experiments, an auxiliary steady LED was used to provide a fixation point. Fading of the peripheral stimulus was avoided by instructing the observers to shift fixation within a narrow range around the fixation point between trials.

The experiment consisted of measuring CFF at each eccentricity over as much as a 4.5-log-unit range of illuminance. The results (Fig. 2) provide a measure of the temporal-response characteristics of the linear component of the visual response.¹ They emphasize the adherence to a straight line in CFF-log-illuminance coordinates, corresponding to the Ferry-Porter law at all eccentricities.

The important feature of the data is the large difference in the slopes of the Ferry-Porter between the foveolar and the peripheral functions, which apply to all illuminance levels. Thus more peripheral stimuli always had higher CFF values, at the same relative illuminance above threshold, than those closer to the fovea. The increase in CFF from 9.86 Hz/decade in the foveola to 22.0 Hz/decade in the far periphery is more than a factor of 2 at all illuminances, just as seen in the Raninen *et al.* data replotted in Fig. 1 above.

METHODOLOGICAL ISSUES

In their discussion, Raninen *et al.* infer that the relative applicability of the various CFF-illuminance laws depends on the experimental conditions tested. While it is hard to dispute such a statement in general, we disagree with the implication that all experimental conditions are equally desirable. We specifically designed our conditions to isolate as much as possible the linear, unadapted response of a single-cone system. (There may be disagreement as to how successful we were, but our subsequent studies are continuing to support the assumptions.^{3,12}) These conditions minimize the confounding effects of light adaptation, flicker adaptation, rod intrusion, intrusion from other cone classes, and variations in temporal properties with retinal eccentricity. From this perspective, any conditions that permit the intrusion of any of these effects may indeed induce departures from the basic temporal function of the underlying transduction events. However, such departures represent the uncontrolled combination of multiple contributory mechanisms and therefore cannot be expected to conform to any homogeneous principle. After a century of study in this field, only excellent fits of hypotheses to data obtained under carefully controlled ex-

perimental conditions should be regarded as definitive. We conclude that there is extensive support for the Ferry-Porter law to high accuracy at all fixed retinal locations when appropriate testing is done.

ACKNOWLEDGMENTS

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